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On colour non-singlet representations of the quark–gluon system at finite temperature

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Abstract. We use a group theoretical technique to project out the partition function for a system of quarks, antiquarks and gluons onto a particular representation of the internal symmetry group $SU(3)$: the colour singlet, colour octet and colour 27-plet, at finite temperature. We do this to calculate the thermodynamic quantities for those representations. We also calculate the change in free energy of the plasma droplet formed from the hot hadronic gas. We find that the size of the droplet in the colour-octet representation is smaller than that in the colour-singlet representations at different temperatures in the vicinity of the critical temperatures of the phase transitions.

1 Introduction

Colour confinement is an experimentally well established property of QCD at temperature $T = 0$. Though it has not been conclusively demonstrated in QCD, it is universally believed to be true. Several model calculations indicate that indeed the $3q$ and $q\bar{q}$ colour-singlet states are bound more than for example the colour-octet, -decuplet representations, etc. [1]. However, one cannot simply throw away the higher-colour representations like the octet as they manifest themselves in specific situations like in multiquark systems [2] etc. Recently the colour-octet contribution has also been shown to be significant during quarkonium production in hadronic collisions [3]. In this paper we study the role of higher representations like the octet, the 27-plet etc. for bulk QGP at finite temperature. Here we shall demonstrate their significance and the new insights that they yield.

It is believed that in QCD the “transition from hadronic matter to quark–gluon matter is a transition from local colour confinement (on the scale of 1 fm) to global colour confinement” [4]. With the mathematical development of the consistent inclusion of internal symmetries in a statistical thermodynamical description of quantum gases [5], the idea was applied to the colour $SU(3)$ group [6,7]. The group theoretical projection technique was used to project out colour-singlet representation for a bulk system consisting of a quark–gluon plasma (QGP) at finite temperature. For $SU(3)$, due to the colour conservation the whole QGP fireball needs to be a colour singlet. The colour-projection technique allowed one to do this in a neat way.

The requirement of the imposition of being a colour singlet for these systems has been found to be of great significance and much work has been done using this technique of colour projection [8–13]. Several interesting results were obtained; one of them was that if one were to compare a colour-unprojected bulk QGP system with a colour-singlet projected QGP system, then important finite-size corrections are introduced [6,7]. These finite-size corrections arising from the imposition of being a colour singlet disappear as the size and/or temperature of the system increases. This was taken to mean that for large-sized QGP systems, which may have been relevant in the early universe, for QCD phase transition scenarios one may automatically assume globally it to be a colour singlet [4] of the system without any significant modifications. This allowed for the possible existence of large-size stable quark stars (which were trivially assumed to be colour singlet [14]) in the early universe QCD phase transition [15–18]. These scenarios continue to dominate the hadronisation ideas in the big bang models [19]. These ideas have also been quite significant in the heavy ion collision scenarios as well.

To understand the role of the colour degree of freedom we use the colour-projection technique [5–13]. In this technique one constructs a generating partition function from which the restricted partition function for any given irreducible representation can be obtained. We give the necessary mathematical details for the colour-projection technique in Sect. 2; the results will be discussed in Sects. 3 and 4, respectively.

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2 Colour-projection technique

The orthogonality relation for the associated characters $\chi_{(p,q)}$ of the (p,q) multiplet of the group $SU(3)$ with the measure function $\zeta(\phi,\psi)$ is

$$\int_{SU(3)} \mathrm{d}\phi\mathrm{d}\psi\zeta(\phi,\psi)\,\chi^{\star}_{(p,q)}(\phi,\psi)\,\chi_{(p',q')}(\phi,\psi) = \delta_{pp'}\delta_{qq'}, \quad (1)$$

where the measure function [7] is given by

$$\zeta(\phi,\psi) = \left[\sin\frac{1}{2}\left(\psi+\frac{\phi}{2}\right)\sin\frac{\phi}{2}\sin\frac{1}{2}\left(\psi-\frac{\phi}{2}\right)\right]^2. \quad (2)$$

Let us now introduce the generating function Z^G by

$$Z^G(T,V,\phi,\psi) = \sum_{p,q}\frac{Z_{(p,q)}(T,V)}{d(p,q)}\chi_{(p,q)}(\phi,\psi), \quad (3)$$

with

$$Z_{(p,q)}(T,V) = \mathrm{tr}_{(p,q)}\left[\exp\left(-\beta\hat{H}_0\right)\right], \quad (4)$$

where $Z_{(p,q)}$ is the canonical partition function. The many-particle states which belong to a given multiplet (p,q) are used in the statistical trace with the free hamiltonian $\hat{H}_0$, $d(p,q)$ is its dimensionality and β is the inverse of the temperature T. The projected partition function $Z_{(p,q)}$ can be obtained by using the orthogonality relation for the characters. Hence the projected partition function for any representation (p,q) is

$$Z_{(p,q)}(T,V) = d(p,q)\int_{SU(3)}\mathrm{d}\phi\mathrm{d}\psi\zeta(\phi,\psi)\,\chi^{\star}_{(p,q)}(\phi,\psi) \times Z^G(T,V,\phi,\psi)\,. \quad (5)$$

The characters of the different representations are as follows:

$$\chi_{(1,0)} = \exp(2\mathrm{i}\psi/3) + 2\exp(-\mathrm{i}\psi/3)\cos(\phi/2), \quad (6)$$

$$\chi_{(0,1)} = \chi^{\star}_{(1,0)}, \quad (7)$$

$$\chi_{(1,1)} = 2 + 2\left[\cos\phi + \cos(\phi/2+\psi) + \cos(-\phi/2+\psi)\right], \quad (8)$$

$$\chi_{(2,2)} = 2 + 2\left[\cos\phi + \cos(3\phi/2)\cos(\phi/2)\right] + 2(1+2\cos\phi)\left[\cos(\phi/2+\psi) + \cos(-\phi/2+\psi) + \cos 2\psi + (1/2)\cos\phi\right]. \quad (9)$$

The expressions of the generating function used in (5) is

$$Z^G(T,V,\phi,\psi) = \mathrm{tr}\left[\exp(-\beta\hat{H}_0 + \mathrm{i}\phi\hat{I}_z + \mathrm{i}\psi\hat{Y})\right], \quad (10)$$

where $\hat{I}_z$ and $\hat{Y}$ are the diagonal generators of the maximal abelian subgroup of $SU(3)$. Our plasma consists of light spin 1/2 (anti) quarks in the (anti-) triplet representation $(0,1)$ and $(1,0)$, respectively, and massless spin one gluons in the octet representation $(1,1)$. Note that the non-interacting hamiltonian $\hat{H}_0$ is diagonal in the occupation-number representation. In the same representation one can write the charge operators $\hat{I}_z$ and $\hat{Y}$ as linear combinations of the particle-number operators. Hence Z^G can be easily calculated in the occupation-number representation. With an imaginary "chemical potential" this is just like a grand canonical partition function for free fermions and bosons. One obtains

$$Z^G_{\mathrm{quark}} = \prod_{q=l,m,n}\prod_k\left[1+\exp(-\beta\epsilon_k - \mathrm{i}\alpha_q)\right] \times \left[1+\exp(-\beta\epsilon_k + \mathrm{i}\alpha_q)\right], \quad (11)$$

$$Z^G_{\mathrm{glue}} = \prod_{g=\mu,\nu,\rho,\sigma}\prod_k\left[1-\exp(-\beta\epsilon_k + \mathrm{i}\alpha_g)\right]^{-1} \times \left[1-\exp(-\beta\epsilon_k - \mathrm{i}\alpha_g)\right]^{-1}. \quad (12)$$

Here the single-particle energies are given as ϵ_k. For the $(1,0)$, $(0,1)$ and $(1,1)$ multiplets, the eigenvalues of $\hat{I}_z$ and $\hat{Y}$ give the expressions for the different angles:

$$\alpha_l = (1/2)\phi + (1/3)\psi, \quad \alpha_m = (-1/2)\phi + (1/3)\psi, \quad \alpha_n = (-2/3)\psi, \quad (13)$$

$$\alpha_\mu = \alpha_l - \alpha_m, \quad \alpha_\nu = \alpha_m - \alpha_n, \quad \alpha_\rho = \alpha_l - \alpha_n, \quad \alpha_\sigma = 0. \quad (14)$$

We neglect the masses of the light quarks. At large volume the spectrum of a single particle becomes quasi-continuous and $\Sigma \ldots \to V/(2\pi)^3\int \mathrm{d}^3k\ldots$ Then one gets

$$Z^G(T,V,\phi,\psi) = Z^G_{\mathrm{quark}}(T,V,\phi,\psi)Z^G_{\mathrm{glue}}(T,V,\phi,\psi), \quad (15)$$

which enables us to obtain the partition function for any representation, i.e. $Z_{(p,q)}$. One may thus obtain any thermodynamical quantity of interest for a particular representation. For example, the energy

$$E_{(p,q)} = T^2\frac{\partial}{\partial T}\ln Z_{(p,q)}, \quad (16)$$

and the free energy

$$F_{(p,q)} = -T\ln Z_{(p,q)}. \quad (17)$$

3 Results

On the basis of $SU(3)$ one expects that due to colour conservation the whole QGP fireball should be a colour singlet. Hence work was done by several groups to impose the condition of being a colour singlet on the system [7,?]. Note that in these calculations perturbative interactions had been neglected. But this may not be a bad approximation especially at high temperatures. The most dramatic consequence of the colour interaction is to cause global colour confinement of the quarks and gluons and this is automatically taken care of by restricting the partition function to colour singlets [4]. This perhaps may amount to the handling of a major chunk of the non-perturbative aspect of the QCD interaction. It was found that

$$E_{(0,0)} = E_0 + E_{\mathrm{corr}}, \quad (18)$$

where E_0 was the unprojected energy (i.e. with no colour restriction whatsoever) given by

$$E_0 = 3a_q VT^4, \tag{19}$$

with $a_q = (37\pi^2/90)$ and $E_{\rm corr}$ was the correction introduced due to the imposition of being a colour singlet. Elze et al. [7,8] have found that $E_{\rm corr}$ was significant only for a finite size, i.e. when $TV^{1/3}$ was small (< 2), and vanished when $TV^{1/3}$ became large (> 2). This would mean that the colour-singlet restriction only affects for a size of say ~ 1.0 fm for $T = 160$ MeV, while for a large size and higher temperatures one need not perform an explicit colour-projection calculation because the consequent corrections are negligible therein. But below we shall show that this is not the whole story.

Here we project out different representations like the colour octet $(1,1)$, the 27-plet $(2,2)$ etc. on the QGP. The idea is that for the ground state one knows that the singlet state is bound and the higher representations are expelled to higher energies [1]. Also for the ground states the role of the higher representations has quite well been studied [2,3]. The point to be emphasised is that the role of the global demand of being a colour singlet at high temperatures is only an assumption and has never been explicitly demonstrated even in a model calculation. Here we would like to study the basis of this assumption and also the role, if any, of higher representations like the octet, the 27-plet, etc. Let us look at the octet, 27-plet etc. projection. For simplicity we take the $\mu = 0$ case with two flavours. We plot in Fig. 1

$$D^{\rm eff}_{(p,q)} = E_{(p,q)}/E_0 = 1 + E^{\rm corr}_{(p,q)}/E_0. \tag{20}$$

Also shown is $D^{\rm eff}_{(0,0)}$ as obtained earlier by other groups [7].

Next we calculate the free energy of the plasma droplet formed in the nucleation process from the hot hadronic gas. Here the fields in the plasma obey the bag boundary conditions, staying inside the plasma droplet [20]. Within the bag model, the change in free energy which is responsible for the nucleation process can be written as

$$\Delta F = -T \ln Z_{(p,q)} + BV + P_{\rm h} V + 4\pi R^2 \sigma, \tag{21}$$

where V is the volume of the plasma droplet with radius R formed in the hot hadronic gas. B is the bag pressure and σ is the surface free energy of the quark–gluon/hadron interface.

If the hadronic gas consists of massless pions only, then the pressure of the hadronic gas $P_{\rm h}$ is given by [21]

$$P_{\rm h} = a_{\rm h} T^4, \tag{22}$$

where $a_{\rm h} = \pi^2/30$.

By calculating the partition function $(Z_{(p,q)})$ numerically for different colour representations (p,q) from (5), we can calculate the change in free energy (ΔF) of the plasma droplet from (21). We plot this change in free energy as a function of the radius (R) of the plasma droplet

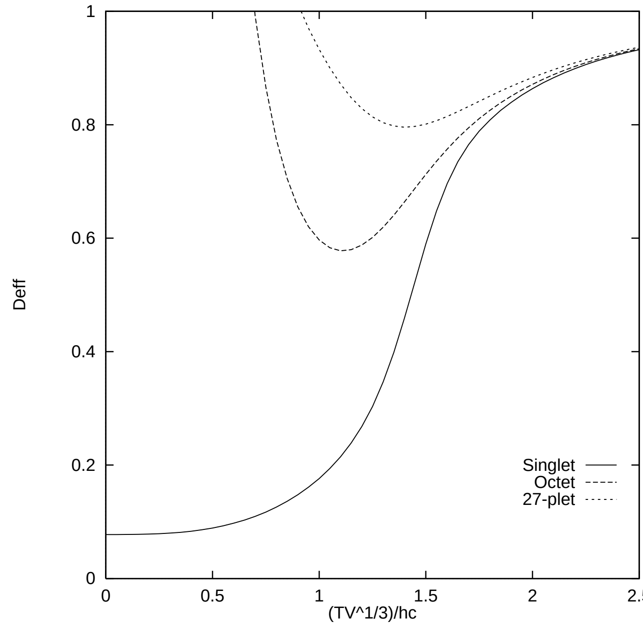

Fig. 1. $D_{\rm eff}$ (see text) for the colour singlet, octet and 27-plet (with two flavours) representation as a function of $TV^{1/3}/\hbar c$

in Fig. 3. We see that for a bag pressure $B^{1/4} = 200$ MeV and $\sigma = 50\,{\rm MeV/fm}^2$ [21], the radius (R) of the plasma droplet for which ΔF has a peak is 1.55 fm for a colour singlet, whereas it is 1.53 fm for the colour-octet representation at a temperature $T = 160$ MeV. At $T = 170$ MeV, the radius (R) is 1.20 fm for the colour singlet, whereas it is 1.15 fm for the colour-octet representation. Similarly for other temperatures as well we find that the radius of the colour-octet system is always smaller than the colour-singlet system.

4 Discussion

It is interesting to note from Fig. 1 that for large values of $TV^{1/3}$ all representations, singlet, octet, 27-plet, etc., approach each other with the unprojected energy. There is nothing which favours the colour-singlet representation over the colour-octet at high temperatures. Note that this result could directly be seen from the expression for $Z_{(p,q)}$, see (4), which for the continuum approximation for a sufficiently large volume and temperature becomes independent of the representation. Hence the energies for each colour representation approach each other for large $TV^{1/3}$. As the negligence of the small perturbative QCD interactions is justified at high enough temperatures, this result seems to be quite model independent.

Note that the quarks and gluons are non-interacting in our model. While this is justified at high temperatures, the neglect of interactions may not be justified at low temperatures. However, as one of the most dramatic effects of the colour interaction is ensuring the imposition that it is a colour singlet on the bulk system, it is already taken

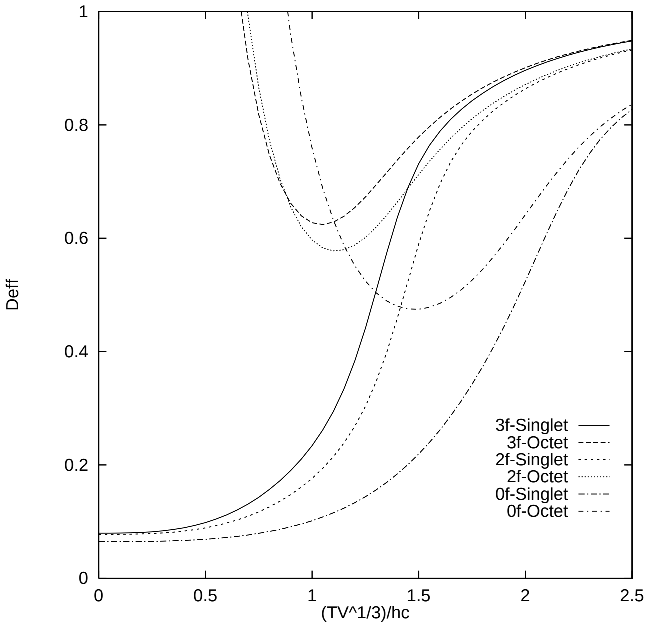

Fig. 2. D_{eff} for the colour singlet and octet representations as a function of $TV^{1/3}/\hbar c$ for different numbers of flavours: 0, 2 and 3

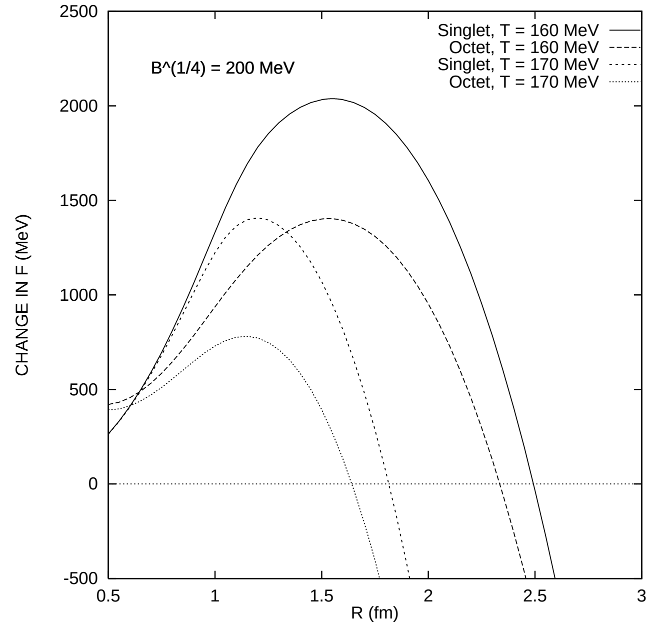

Fig. 3. Change in free energy (ΔF) for the baryon free plasma droplet for the colour singlet and octet representations as a function of the radius (R) of the droplet at different temperatures. The bag pressure is $B^{1/4} = 200\,\text{MeV}$ and the surface free energy $\sigma = 50\,\text{MeV/fm}^2$

care of in our model [4]. So perhaps a significant portion of the non-perturbative effect may already be there in our model calculations.

From Fig. 1 note that for small $TV^{1/3}$ values the octet and the 27-plet energies shoot up. Though gauge interactions are believed to be essential to show confinement in QCD, what we note here is that our projection technique at even low temperatures is able to discriminate between the singlet and the octet states etc. Figure 2 gives a $\mu = 0$ result for zero flavour, two flavour and three flavour for the $(0, 0)$, $(1, 1)$ representations. This shows a similar behaviour of the energy states with colour-singlet and -octet representations for different flavoured systems. The fact that the colour-singlet representation gets favoured over the octet representation etc. at low temperature group theoretically indicates that for composite systems $SU(3)$ is a good symmetry.

We find that when the baryon free quark–gluon plasma droplets are formed in the nucleation process from the hot hadronic gas, the size of the droplet is smaller for the colour-octet representations of $SU(3)$ compared to the size of the colour-singlet droplet. This is observed to be true for different temperatures in the vicinity of the quark–hadron phase transition temperature. So being smaller in size, these colour-octet droplets could dress themselves to produce the global colour-singlet QGP droplets at high temperatures.

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